B.SC. Sem-IV

TOPIC – Charpit Method

Que 1. Solve $px+qy=pq$ by applying Charpit's method.

Sol^n:– Here $F(x,y,z,p,q) = px+qy-pq=0$

So that $\frac{\partial F}{\partial x} = p \cdot \frac{\partial F}{\partial y} = q \cdot \frac{\partial F}{\partial z} = 0$

$$\frac{\partial F}{\partial p} = x-q \text{ and } \frac{\partial F}{\partial q} = y-p$$

Hence Charpit auxiliary equations are

$$\frac{dp}{\frac{\partial F}{\partial x}+p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y}+q\cdot\frac{\partial F}{\partial z}} = \frac{dz}{-p\frac{\partial F}{\partial p}-q\frac{\partial F}{\partial q}} = \frac{dx}{-\frac{\partial F}{\partial p}} = \frac{dy}{\frac{-\partial F}{\partial q}} = \frac{\partial F}{0}$$

$$\Rightarrow \frac{dp}{p+p\cdot 0} = \frac{dq}{q+q\cdot 0} = \frac{dz}{-p(x-q)-q(y-p)} = \frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dF}{0}$$

$$\Rightarrow \frac{dp}{p} = \frac{dq}{q} = \frac{dz}{-px-qy+2pq} = \frac{dx}{-(x-q)} = \frac{dy}{-(y-p)} = \frac{dF}{0}$$

Taking first two, we get $\frac{dp}{p} - \frac{dq}{q} = 0$

integrating $\log p - \log q = \log a \Rightarrow \log\left(\frac{p}{q}\right) = \log a \Rightarrow \frac{p}{q} = a \quad \therefore p = aq$

Putting $p = aq$ in the given equation, we get

$$aqx+qy = aq^2 \Rightarrow q(ax+y) = aq^2 \Rightarrow q = \frac{y+ax}{a}$$

$$p = aq = a\left(\frac{y+ax}{a}\right) = y+ax$$

Now $dz = pdx+qdy \quad \therefore dz = (y+ax)dx + \left(\frac{y+ax}{a}\right)\cdot dy$

$$\Rightarrow dz = (y+ax)\left(dx+\frac{dy}{a}\right) \Rightarrow adz = (y+ax)(adx+dy)$$

$\therefore$ integrating $az = \frac{(y+ax)^2}{2} + b$ (Constant) —— (1)

Singular Integral: differentiating (1) w.r to a and be we get

$$z = \frac{2(y+ax)}{2} \cdot x + 0 \Rightarrow z = x(y+ax)$$

and $0 = 1$

Hence there is no singular integral

General integral: writing $b = \phi(a)$, we get $az = \frac{(y+ax)^2}{2} + \phi(a)$ ——— (2)

Differentiating w.r.to a we get $z = x(y+ax) + \phi(a)$

The 'a' eliminate of (1) and (2) will be the general Integral.

Que:-2. Solve $p = (qy+z)^2$

Sol^n:- Here $F(x, y, z, p, q) = p - (qy+z)^2 = 0$

So that $\frac{\partial F}{\partial x} = 0$ $\frac{\partial F}{\partial y} = -2(qy+z)q$. $\frac{\partial F}{\partial z} = -2(qy+z)$. $\frac{\partial F}{\partial p}$

$= -2(qy+z)y$

Hence Charpit's auxiliary equations are.

$$\frac{dp}{\frac{\partial F}{\partial x} + p\frac{\partial F}{\partial z}} = \frac{dq}{\frac{\partial F}{\partial y} + q \cdot \frac{\partial F}{\partial z}} = \frac{dz}{-p \cdot \frac{\partial F}{\partial p} - q\frac{\partial F}{\partial q}} = \frac{dx}{-\frac{\partial F}{\partial p}} = \frac{dy}{-\frac{\partial F}{\partial q}}$$

$$= \frac{dp}{0 - 2p(qy+z)} = \frac{dq}{2q(qy+z) - q \cdot 2(qy+z)}$$

$$= \frac{dz}{-p \cdot 1 + q \cdot 2y(qy+z)} = \frac{dx}{-1} = \frac{dy}{2y(qy+z)}$$

Taking the first and fifth fraction, we have

$$\frac{dp}{-p} = \frac{dy}{y} \Rightarrow \frac{dp}{p} + \frac{dy}{y} = 0 \Rightarrow \log p + \log y = \log a$$

$$\Rightarrow \log(py) = \log a \Rightarrow py = a \therefore p = \frac{a}{y}$$

Now putting $p = \frac{a}{y}$ in the given equation, we get

$$\Rightarrow y \cdot dz - a\,dx + \frac{\sqrt{a}}{\sqrt{y}} \cdot dy - z\,dy \Rightarrow y\,dz + z\,dy = a\,dx + \frac{\sqrt{a}}{\sqrt{y}} \cdot dy$$

integrating $yz = ax + 2\sqrt{a}\sqrt{y} + b$

which is the Complete integral.